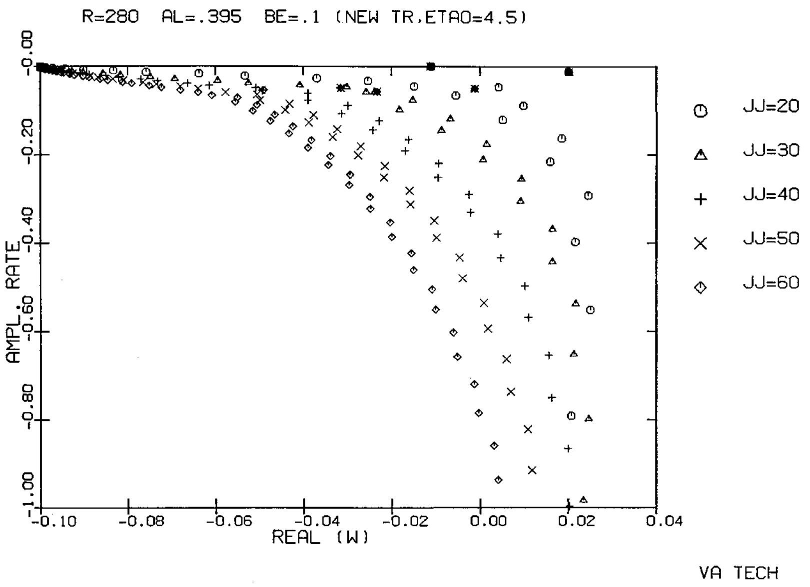

Fig. 1 Eigenvalue spectra for the increasing number of total collocation points: $R=280$, $\alpha=0.395$, $\beta=0.1$.

better than the other transformation, and does not have any convergence problem. Therefore, it is more accurate and useful than the other two transformations considered here.

In this paper, the critical Reynolds number is sought. Therefore, only the principal eigenvalue is of interest. For this purpose, a local search method is employed by using start-up values from the global computations. Details of this process are given by Sahin.[10]

Results and Conclusions

The critical Reynolds number for the rotating disk flow is investigated by using a Chebyshev spectral method. Two algebraic and one exponential transformations are used for mapping the physical space to the spectral space. One of the algebraic transformations is found to be best among the three, partially due to having a resolution of collocation points closer to the wall. Although it appears very efficient in some cases, the exponential transformation is found to have the problem of inconsistency.

In Fig. 1, the effect of J on the eigenvalue spectrum is demonstrated. Increasing J causes more discrete eigenvalues separate from the continuous spectrum. Apparent pairing of the eigenvalues is due to having a system of two differential equations.

By using a local search method, the critical Reynolds number (R_c) is found to be 285, as explained in Ref. 10. This result is consistent with the result of Ref. 4 ($R_c=287$) and in the more recent work of Malik[11] ($R_c=285.36$) that is obtained by a Newton-type iteration. As indicated,[11] this is in excellent agreement with experiments of Wilkinson and Malik.[12] The experiments, however, are not based on controlled normal-mode-type disturbances. Despite success of the linear theory, discrepancies, such as number of vortices observed, still exist between the experiment.

Acknowledgments

The author is indebted to Dr. Thorwald Herbert of Virginia Polytechnic Institute and State University for his support and encouragement.

References

[1]Gregory, N., Stuart, J. T., and Walker, W. S., "On the Stability of Three-Dimensional Boundary Layers with Application to the Flow Due to a Rotating Disk," *Philosophical Transactions of Royal Society*, Vol. 248, 1955, pp. 155–199.

[2]von Karman, T., NACA Technical Memorandum 1092, 1921.

[3]Smith, N. H., NACA TN 1227, 1947.

[4]Malik, M. R., Wilkinson, S. P., and Orszag, S. A., "Instability and Transition in Rotating Disk Flow," *AIAA Journal*, Vol. 19, Sept. 1981, pp. 1131–1138.

[5]Mack, L. M., "The Wave Pattern Produced by a Point Source on a Rotating Disk," AIAA Paper 85-0490, Jan. 1985.

[6]Carnahan, B., Luther, H. A., and Wilkes, J. O., *Applied Numerical Methods*, Wiley, New York, 1969, pp. 178–196.

[7]Fox, L. and Parker, I. B., *Chebyshev Polynomials in Numerical Analysis*, Oxford Univ. Press, 1968, pp. 60–62.

[8]Grosch, C. E. and Orszag, S. A., "Numerical Solution of Problems in Unbounded Regions: Coordinate Transforms," *Journal of Computational Physics*, Vol. 25, 1977, pp. 273–296.

[9]Spalart, P. R., "A Spectral Method for External Viscous Flows," Proceedings of the AMS Conference on Fluids and Plasma, Geometry and Dynamics, June 1983.

[10]Sahin, I., "The Application of Various Coordinate Transformations for Rotating Disk Flow Stability," AIAA Paper 86-0500, Jan. 1986.

[11]Malik, M. R., "The Neutral Curve for Stationary Disturbances in Rotating-Disk Flow," *Journal of Fluid Mechanics*, Vol. 164, 1986, pp. 275–287.

[12]Wilkinson, S. P. and Malik, M. R., "Stability Experiments in the Flow Over a Rotating Disk," *AIAA Journal*, Vol. 23, April 1985, pp. 588–595.

Supersonic Base Pressure and Lipshock

Eric C. Magi* and Sudhir L. Gai†
University College, University of New South Wales, Australian Defence Force Academy, Australia

Introduction

MANY theories have been proposed to describe base flows. Probably the best known theory is the Chapman-Korst[1] model. This model and its subsequent derivatives are

Received Dec. 11, 1986; revision received June 1, 1987.

*Teaching Fellow, Dept. of Mechanical Engineering.
†Lecturer, Dept. of Mechanical Engineering.

able to predict steady base pressure, with good approximation, for moderate supersonic Mach numbers (≥ 1.6) and thin separating turbulent boundary layers. They show only a qualitative agreement, however, in the variation of base pressure, as a function of separating boundary-layer thickness. The main problem appears to be the formulation of an adequate reattachment criteria.

Tanner has circumvented this problem by using Oswatitsch's theorem, relating drag to rate of entropy increase in the flow. In his supersonic base pressure theory, Tanner[2] equates real flow with an inviscid flow with same base pressure. Base pressure is obtained by equating the rate of entropy increase, due to the reattaching shear layer, with the difference in rate of entropy increase in the wake shocks of the two flows.

Tanner has assumed that the influence of the separating boundary layer on base pressure is manifested in modification of momentum thickness of the reattaching shear layer. The theoretical values of base pressure are in good agreement with the experiment for Mach numbers in the range of $M = 1.1$ to 3, and separating boundary layers with momentum thicknesses up to $\theta/h = 0.85$, where θ is momentum thickness,[2,3] and h is base height.

All of the above base flow theories, however, including that of Tanner,[2] do not take explicitly into account the effect of lipshocks emanating from the base region. Hama[4] has shown conclusively that lipshocks can be of considerable strength. Consequently, influence of lipshocks on the near wake, and hence base pressures, can be significant. This raises the issue of why Tanner's theoretical values are in good agreement with experimental data.

Base Pressure Calculations

Following Tanner's notation, the equation to be satisfied is

$$K_1(M_\infty)\,\frac{\gamma}{2}\,M_\infty^2 H = F_2(M_2,\beta)\,H^* \tag{1}$$

where $1/2K_1(M_\infty)$ is the nondimensionalized momentum thickness of the reattaching shear layer and the function $F_2(M_2,\beta)$ is ratio of total pressures before and after wake shock $[= \ln\,(P_{0\infty}/P_{04})]$. H^* is the height of the "shear layer-affected" wake shock and H is the reattaching shear-layer thickness.

It is assumed that the expansion fan is isentropic and the pressure gradient across the shear layer is negligible. Hence, base pressure is determined from the deflection angle of the separating flow. The value of H^*/H is assumed constant. It is determined by equating Eq. (1) with a measured base pressure at the condition $M_\infty = 1.73$ with a small separating boundary layer. Thus, the value of H^*/H is equal to 7.37.

Using Tanner's theory, the author has made calculations that are shown in Figs. 1 and 2. Tanner's calculations are also included in these figures along with experimental data.

Results and Discussion

Lipshocks and the H^*/H Parameter

The lipshock is an important viscous phenomena. Behavior of lipshocks is closely linked to the state of the separating and reattaching boundary layers in a step or wedge flow. Appearance of lipshock is due to an overexpansion of flow at the separation edge. The expansion fan is terminated by the lipshock, which then recompresses the flow to base pressure. Shape, location, and strength of the lipshock are functions of Mach number, Reynolds number, and geometry of the base corner.

Consider a flow regime where a lipshock forms at a low Mach number and high Reynolds number (typically 5×10^5 based on freestream conditions and chord length), where the separating boundary layer is laminar. Lipshock forms close to

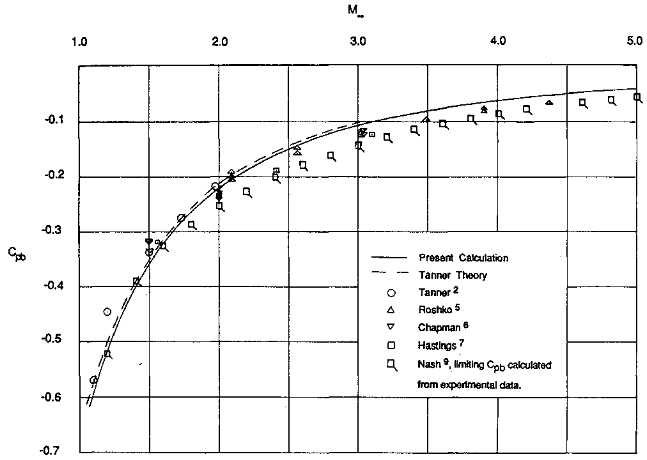

Fig. 1 Base pressure coefficient as a function of freestream Mach number.

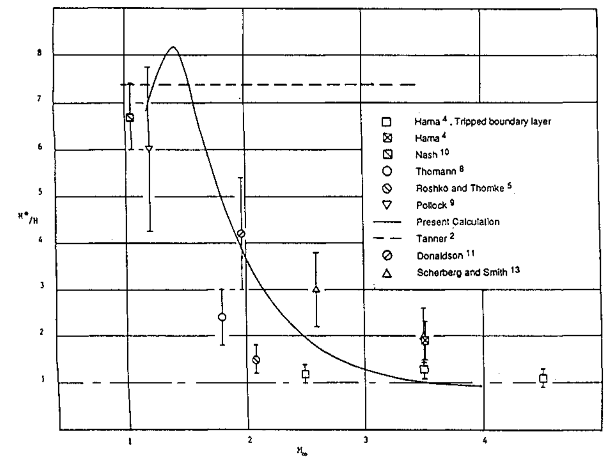

Fig. 2 Estimates of H^*/H from flow visualization and present calculation using base pressure coefficient from Nash and Eq. (1).

the separation point in the free shear layer and extends out into the expansion fan. (See Fig. 1, Ref. 4.) As the Reynolds number increases, the over-expansion becomes more severe and the lipshock tends to rotate downward towards the axis in the case of a wedge, or toward the wall in the case of a rearward facing step.

Hama noted that by tripping the separating boundary layer, the lipshock remained embedded in the free shear layer. Lipshocks can be clearly seen in shadowgraphs (Fig. 9, Ref. 4) and are detectable by pitot pressure measurements. A lipshock of significant strength would thus affect the wake profile. Shadowgraphs in Hama's paper seem to suggest that the lipshock, which is completely embedded in the shear layer, is somewhat diffused by stresses associated with a turbulent-free shear layer. It would seem reasonable then to assume that the parameter H^*/H is also a function of the Mach and Reynolds numbers, because it is influenced by lipshock-wake shock interaction. It is generally noted that strength of the wake shock is considerable in the vicinity of the wake neck. Estimates of H^*/H taken from schlieren photographs and shadowgraphs of earlier investigations were made assuming that H is of the order of the wake neck.

Nash[9] compiled the most probable values of C_{pb} with thin separating boundary layers taken from experimental data. Estimates of H^*/H were calculated using these values of C_{pb} and Eq. (1). The results are shown in Fig. 2. The available experimental data is also included.

The calculations show a trend whereby the value of H^*/H decreases as Mach numbers increase. Also, the value of H^*/H for base flows with turbulent separating boundary layers are of the order of unity and show a trend similar to, but away from, laminar ones. Results shown in Fig. 2 would therefore suggest that Tanner's assumption that H^*/H being constant, is at best approximate, and his suggested value of $H^*/H \approx 7$ would yield correct values of base pressures only at low supersonic Mach numbers.

If we assume the lipshock to be of significant strength, then flow geometry of the nearwake is necessarily altered. Consequently, an effective shift of the wake shock toward the base results in a decrease of entropy flux due to wake shock in the real flow, as opposed to that in a purely inviscid flow. This forward shift of the wake shock may be perceived as a measure of entropy flux of shear layer and lipshock.

This is still consistent with the Oswatitsch's theorem relating entropy flux to drag. The schlieren photograph of Fig. 3 in Ref. 5 clearly shows a forward translation of wake shock in the real flow. An analogous situation is presented in incident shock/flat plate boundary-layer interactions where the reflected shock wave shifts upstream of the equivalent inviscid position.

It seems plausible, therefore, that there are two mechanisms operating in the forward shift of wake shock in the base flow: (1) the flow geometry is changed by overexpansion of the flow, and (2) the reattaching shear layer-lipshock interaction.

Conclusion

Tanner's theory neglects lipshocks, but he has implicitly accounted for their influence in the ratio H^*/H by equating Eq. (1) with an experimental result. This would explain why Eq. (1) gives a reasonable approximation to $C_{pb}(M_\infty)$. Yet there seems to be no theoretical justification for this. In fact, for turbulent base flow, it seems more appropriate to consider that H^*/H is of the order unity, and available experimental data support this view.

References

[1]Korst, "A Theory for Base Pressures in Transonic and Supersonic Flow," *Journal of Applied Mechanics,* Vol. 23, No. 4, Dec. 1956, pp. 593–600.

[2]Tanner, M., "Steady Base Flows," *Prog. Aerospace Science,* Vol. 21, 1984.

[3]Tanner, M., "Boundary Layer Thickness and Base Pressure," *AIAA Journal,* Vol. 24, Dec. 1985, pp. 1987–1989.

[4]Hama, F. R., "Experimental Studies on the Lipshock," *AIAA Journal,* Vol. 6, Feb. 1968, pp. 212–219.

[5]Roshko, A. and Thomke, G. J., "Observations of Turbulent Reattachment Behind an Axisymmetric Downstream-Facing Step in Supersonic Flow," *AIAA Journal,* Vol. 4, June 1966, pp. 975–980.

[6]Chapman, D. R., Wimbrow, W. R. and Kester, R. H., "Experimental Investigations of Base Pressures on Blunt-Trailing-Edge Wings at Supersonic Velocities," NACA Technical Note, 2611, Jan. 1952.

[7]Hastings, R. C., "Turbulent Flow Past Two-Dimensional Bases in Supersonic Streams," RAE Technical Note, Aero 2931, Dec. 1963.

[8]Thomann, H., "Measurements of Heat Transfer and Recovery Temperature in Regions of Separated Flow at a Mach Number of 1.8," FFA Rept. 82, Nov. 1958.

[9]Nash, J. F., "A Discussion of Two-Dimensional Turbulent Base Flows," *ARC R&M,* No. 3468, July 1965.

[10]Nash, J. F., "A Review of Research on Two-Dimensional Turbulent Base Flows," *ARC R&M,* No. 3323, March 1962.

[11]Donaldson, I. S., "On the Separation of a Supersonic Flow at a Sharp Corner," *AIAA Journal,* Vol. 5, June 1967, pp. 1086–1088.

[12]Pollock, N., "Some Effects of Base Geometry on Two-Dimensional Base Drag at Subsonic and Transonic Speeds," ARL, Aero Note 316, Australian Defence Scientific Service, Oct. 1969.

[13]Scherburg, M. G. and Smith, H. E., "Experimental Study of Supersonic Flow over a Rearward Facing Step," *AIAA Journal,* Vol. 5, Jan. 1967, pp. 51–56.

Source Term Decomposition to Improve Convergence of Swirling Flow Calculations

D. S. Jang* and S. Acharya†

Louisiana State University, Baton Rouge, Louisiana

Introduction

SWIRLING flows are commonly encountered in aerospace engineering applications. Typical examples are flow in a gas turbine combustor, trailing vortex flow behind an aircraft, etc. The governing differential equations for these flows are the continuity and the Navier–Stokes equation, and in the numerical calculation of such flows, two factors have an important effect on the convergence characteristics of the numerical scheme. The first is the pressure–velocity coupling and the method used to resolve this coupling. The second is the coupling between the momentum equations through their respective source terms. Although the treatment of the pressure–velocity coupling has been systematically studied in the literature,[1–5] methods to resolve efficiently the source term coupling between the momentum equations have not been given the same degree of attention. However, in swirling flows, the source terms, and in particular, the centrifugal term $\rho w^2/r$ in the radial momentum equation become important, and if the source term coupling is not properly addressed, rather poor convergence characteristics of the numerical scheme is noted. The objective of this technical note is to present a source term decomposition technique for the radial momentum equation which greatly enhances the convergence properties of the calculation method and thus reduces the computational time.

Governing Equations

The governing differential equations for steady, laminar, axisymmetric, incompressible flow can be written as

Continuity:

$$\frac{\partial u}{\partial x}+\frac{1}{r}\frac{\partial (rv)}{\partial r}+\frac{1}{r}\frac{\partial w}{\partial \theta}=0 \tag{1}$$

Axial momentum:

$$\rho \mathbf{u}\cdot\nabla u=-\frac{\partial p}{\partial x}+\mu\left[\frac{\partial^2 u}{\partial x^2}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)\right] \tag{2}$$

Radial momentum:

$$\rho \mathbf{u}\cdot\nabla v=-\frac{\partial p}{\partial r}+\mu\left[\frac{\partial^2 v}{\partial x^2}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r}\right)\right] -\frac{\mu v}{r^2}+\frac{\rho w^2}{r} \tag{3}$$

Tangential momentum:

$$\rho \mathbf{u}\cdot\nabla w=\mu\left[\frac{\partial^2 w}{\partial x^2}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right)\right]-\frac{\mu w}{r^2}-\frac{\rho v w}{r} \tag{4}$$

Received April 20, 1987; revision received Aug. 30, 1987.

*Graduate Student, Mechanical Engineering Department.

†Associate Professor, Mechanical Engineering Department.